Reading: Holmes.Intro
Model: Source Text
Problem Type: Offsets, linear predictors, and rating tables

Given

An actuary is building a basic homeowners pure premium GLM using the following variables:
Age of Home which takes integer values between 0 and 10 (inclusive) but is modeled as a continuous variable.
NoFireExtinguisherInd which is 1 if **no** fire extinguisher is present in the home and 0 otherwise.
The base level for *NoFireExtinguisherInd* is 0 (more homes have a fire extinguisher than do not).

The actuary is using a Tweedie distribution with the log link function.

The results of their GLM are shown below.

	Estimate
(Intercept)	4.605
Age of Home	0.010
NoFireExtinguisherInd:1	0.182

You may assume all variables are statistically significant and should be included in the model.

The actuary has performed a separate loss elimination ratio analysis to price their deductible offerings:

Deductible	Discount / Surcharge
$500	15%
$1,000	0%
$2,000	-10%

Find

a.) What is the value of the linear predictor for a home that is 2 years old without a fire extinguisher and having a $500 deductible?

b.) Construct rating tables for the *Age of Home* and *NoFireExtinguisherInd* rating variables.

c.) Briefly describe two potential weaknesses of the actuary's GLM in relation to the *Age of Home* rating variable.

Solution Holmes_Factors (Solution 1)

a.) First we should translate the deductible discount/surcharge into a table of relativities and then adjust them to match the scale of the GLM.

Deductible	Relativity	Offset ln(Rel)
$500	1.150	0.1398
$1,000	1.000	0.0000
$2,000	0.900	-0.1054

Linear predictor $g(\mu_i) = \beta_0 + \beta_1 \cdot Age\ of\ Home_i + \beta_2 \cdot NoFireExtinguisherInd_i + Offset_i$

β_0 = 4.605
β_1 = 0.010
β_2 = 0.182

$g(\mu_i)$ = 4.9468 = 4.605 + 0.010 * 2 + 0.182 * 1 + 0.1398

b.) The inverse of the log link function, ln(μ), is exp(μ).

$$\mu_i = \exp(\beta_0 + \beta_1 \cdot Age\ of\ Home_i + \beta_2 \cdot NoFireExtinguisherInd_i + Offset_i)$$
$$= e^{\beta_0} \cdot \left(e^{\beta_1}\right)^{Age\ of\ Home_i} \cdot \left(e^{\beta_2}\right)^{NoFireExtinguisherInd_i} \cdot e^{Offset_i}$$

The multiplicative relativities for *Age of Home* and *Fire Extinguishers* are then

Age of Home	Relativity
0	1.000
1	1.010
2	1.020
3	1.030
4	1.041
5	1.051
6	1.062
7	1.073
8	1.083
9	1.094
10	1.105

Fire Extinguisher	Relativity
Yes	1.000
No	1.200

c.) Two potential weaknesses are:

- Age of Home was not logged so it doesn't match the scale of the link function. This is why we end up with Age of Home as a power rather than a direct multiplication.
- Age of Home only takes discrete integer values yet is modeled as a continuous variable via a first-order polynomial. It may be better to treat it as a categorical variable and possibly group some of the ages.